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Computation of the plastic hardening curve based on a static tensile test

Wyznaczanie krzywej umocnienia plastycznego na podstawie statycznej próby rozciągania

MATEUSZ ŚLIWKA *

Numerical calculations, concerning, among other things, metal forming, based on the finite element method, require a description of the relationship between stress and strain in the entire load range. After crossing the yield point, the strain hardening is appearing. This phenomenon can be described by the plastic hardening curve. This paper presents the method of determining the strain hardening curve by means of power function approximation. The load and displacement data recorded in the tensile test using the MTS measurement system were used to determine this curve.

KEYWORDS: uniaxial tensile test, strain hardening curve, optimization

Numerical calculations in engineering tasks, especially in the field of plastic working, require description of the elastic and non-elastic characteristics of the material. While in the case of a linear elastic material it is sufficient to determine two material constants (*E* and *v*), in the plastic domain it is necessary to determine the full relation between stress and strain [4]. It can be obtained on the basis of a static tensile test – as a result of the description of the curve of plastic strengthening by means of an appropriate approximation. Many ways of describing plasticizing stresses are known. Plastic flow curves can be classified into one of six groups [1]:

- group I – including functions of the $\sigma = f(\varepsilon)$ type, giving the dependence of stresses on strains;

group II – including functions of the form $\sigma = f(\varepsilon, \dot{\varepsilon})$, which additionally take into account the strain speed $\dot{\varepsilon}$;

- group III – including functions of the type $\sigma = f(\varepsilon, \dot{\varepsilon}, T)$, which in relation to group II are enriched by the temperature value of the shaping process *T*;
- group IV – including functions of the form $\sigma = f(\varepsilon, \dot{\varepsilon}, \sigma_w)$, which in relation to group II additionally introduce a parameter describing the internal state of the material σ_w;
- group V – including functions of the type $\sigma = f(\varepsilon, \dot{\varepsilon}, T, t)$, whose variables are: strain ε, strain speed $\dot{\varepsilon}$, temperature *T* and time *t*;
- group VI – including the functions of group III, which in the subsequent stages of the test take into account a change in the orientation of the main directions h_ε; functions take the form $\sigma = f(\varepsilon, \dot{\varepsilon}, T, h_\varepsilon)$.

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The curves of groups I–III are most often used to describe the curve of plastic strengthening.

This article presents the method of determining the flow curve, classified in group I, based on the tensile test of steel flat specimen in accordance with PN-EN ISO 6892-1. Experimental tests were carried out using a measuring device consisting of a MTS hydraulic cylinder (equipped with a force sensor and a displacement sensor) and an adapter for tensile test on flat specimen, made according to the author's design (fig. 1). The range of loads carried out by the actuator is within ±25 kN.

Two EPSILON extensometers were used to measure longitudinal and transverse deformations. The extensometer measuring longitudinal strain can work in the range of +20 mm/–8 mm, and the extensometer for transverse deformation – in the range of ±5 mm.

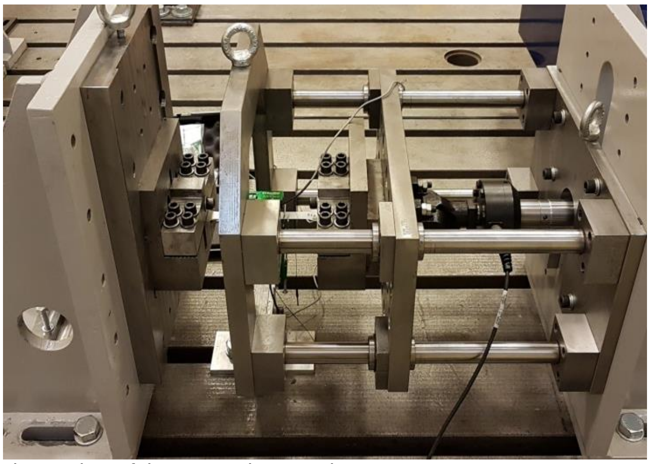

Fig. 1. View of the measuring stand

Experimental research

The tests were carried out according to the author's tensile test procedure, created in the MTS TestSuit program. Fig. 2 shows a dog bone specimen after an endurance test, on which the extensometers for measurements of longitudinal and transverse deformations were still attached. A crack appears on the damaged specimen, running at an angle of about 45° to the direction of the main stresses, i.e. towards the maximum tangential stresses [2].

As a result of the tests, the dependence of stress on strain over the entire load range was obtained (fig. 3). Experimental material parameters are summarized in tab. I.

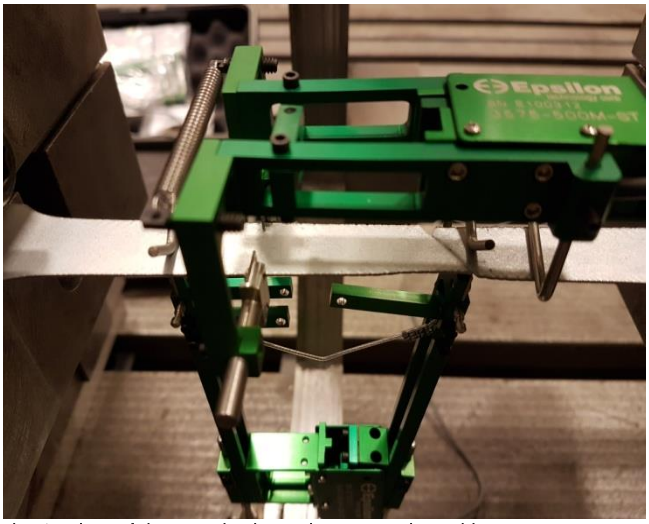

Fig. 2. View of the attached specimen together with extensometers

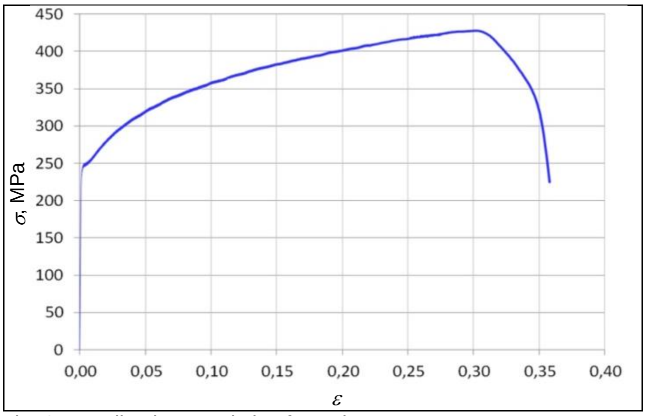

Fig. 3. Tensile characteristic of specimen

TABLE I. Material parameters determined in the tensile test

E, MPa	R_{05}, MPa	R_{02}, MPa	R_m, MPa
223746	238,97	245,36	408,0

E – Young's modulus, R_{02} – offset yield strength, R_{05} –elastic limit, R_m – ultimate tensile strength

In order to carry out numerical simulation of sheet metal forming processes in FEA-based programs, it is necessary to give the σ–ε dependence in the plastic domain, which entails entering into the calculation program the coordinates of the points from the strengthening curve. This is a very labor-intensive operation, because to obtain a faithful image of dependencies in the non-linear range, a large number of data must be manually entered into the calculation program. An alternative solution is to determine a function that approximates the course of the strengthening curve. In the procedure used in the tensile test all the values needed to determine the function approximating the experimental curve were recorded.

In the plastic domain, the strengthening curve can be described, for example, with the Hollomon fortification rule [3, 7], which is a simple power function of the form:

$$\sigma = K\varepsilon^n \tag{1}$$

where: *ε* – real strain; *K, n* – parameters determined from the power approximation of the test results in the range from the offset yield strength $R_{0,2}$ to the ultimate tensile strength R_m according to the PN-ISO 10275 standard.

In order to determine the coefficients *K* and *n*, the formula (1) is transformed, which consists in logarithmization of pages, which leads to obtaining a linear function in logarithmic coordinates:

$$\mathrm{Ln}\,\sigma = \ln K + n \ln \varepsilon \tag{2}$$

where: *n* – slope of the linear function, *K* – y-intercept of the linear function.

The exponent of the strengthening *n* was determined from the formula:

$$n = \frac{\sum_{j=1}^{m}\left(\ln \varepsilon_j - \bar{\varepsilon}_{śr}\right)\left(\ln \sigma_j - \bar{\sigma}_{śr}\right)}{\sum_{j=1}^{m}\left(\ln \varepsilon_j - \bar{\varepsilon}_{śr}\right)^2} \tag{3}$$

where: $l_{n\varepsilon j}$, $l_{n\sigma j}$ – values calculated on the basis of logarithmized values of strains ***ε*** and stresses ***σ***.

The summation in formulas (3) and (4) takes place in the range from *j* = 1 to *m*, i.e. from $R_{0.2}$ to R_m. On the other hand, the values in formula (3) were determined as follows:

$$\bar{\varepsilon}_{śr} = \frac{1}{m}\sum_{j=1}^{m} \ln\varepsilon_j \qquad \bar{\sigma}_{śr} = \frac{1}{m}\sum_{j=1}^{m} \ln\sigma_j \tag{4}$$

where: *i* – average values of strains and stresses ranging from $R_{0.2}$ to R_m.

The factor *K* is calculated from the dependence:

$$K = \exp(\bar{\sigma}_{śr} - n\bar{\varepsilon}_{śr}) \tag{5}$$

From the formulas (3)–(5), the stresses σ_H were determined. The graph (fig. 4) shows the experimental curve and the approximation curve according to the Hollomon's rule.

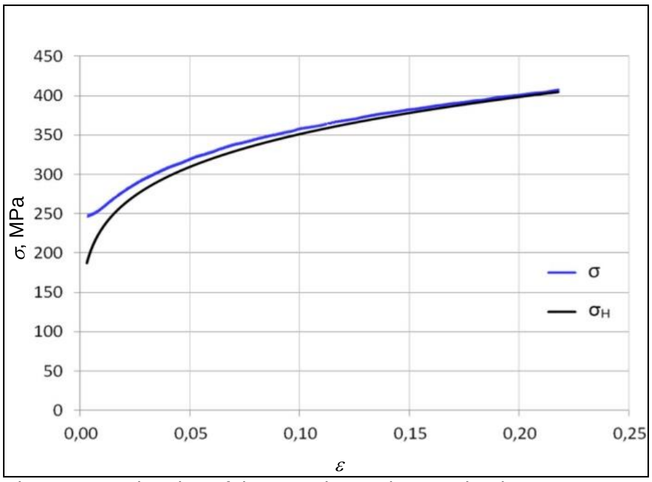

Fig. 4. Approximation of the experimental strengthening curve

At the beginning of the strengthening curve, the stress value σ_H [5] jumps, and the adjustment to the experimental curve is not satisfactory, which is confirmed by the coefficient of determination R^2 equal to 0.734.

In order to improve the fit of the curves, a modified concept of determining the strengthening curve was used, i.e. the strengthening rule according to Krupkowski [8].

According to it, in the stress formula σ_K the strain is divided into two values:

$$\sigma_K = K(\varepsilon_0 + \varepsilon_p)^n \tag{6}$$

where: ε_0 – initial strain, ε_p – plastic strain.

If the value of ε_0 in formula (6) is to accept strains corresponding to the offset yield point, then the Hollomon formula is obtained [6].

The stress equation formula σ_K contains three parameters: K, ε_0 and n. To find their values, for which the best fit of the Krupkowski curve will occur to the experimental results, the optimization task was defined using the solver built into the Excel spreadsheet. The approximate approximation parameters – K, n and initial strain ε_0 – were used in the optimization task as decision variables. The measure of the fit approximation $\sigma_K(\varepsilon)$ to the experimental curve $\sigma(\varepsilon)$ is the value calculated as the sum of the squares of the difference between the stress value σ_K and σ, which is denoted as δ. The objective function in the task is to minimize this value. The parameter has been specified in the range from R_{02} to R_m:

$$\delta = \sum_{j=1}^{m} (\sigma_{Kj} - \sigma_j) \tag{7}$$

where: σ_K – stress determined from the Krupkowski formula, σ – the value determined on the basis of the force recorded during the tensile test.

As a result of the optimization, modified values of decision variables were obtained, which much better fit the σ_K approximation curve to the experimental results.

Fig. 5 shows the experimental curve and the approximation curve according to Krupkowski's rule.

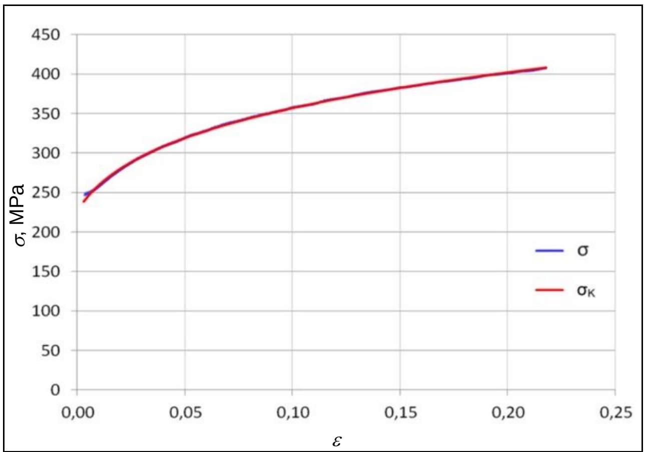

Fig. 5. Approximation of the experimental strengthening curve according to Krupkowski's rule

Thanks to the application of Krupkowski's rule and solving the optimization task, the R^2 coefficient of determination was obtained equal to 1 with the accuracy of three significant digits, which indicates a very good matching of the $\sigma_K(\varepsilon)$ function to the $\sigma(\varepsilon)$ experimental curve. The δ value of this curve is 5696, which is ten times less than in the case of the Hollomon rule.

Tab. II lists the parameters K, n, ε_0, δ and coefficients of determination determined for the described rules of strengthening.

TABLE II. Parameters of the plastic strengthening curve

Principle	K, MPa	n	ε_0	δ	R^2
σ_H	504	0,1486	–	53103	0,734
σ_K	535	0,1827	0,0089	5696	1,000

Conclusions

The mechanical properties of the sheet supplied by the manufacturer should be within the limits specified for a given type of steel. Even a slight difference in mechanical properties can affect the production of defective, incorrectly shaped emboss. For this reason, it is very important to carry out the control tests on a regular basis during the punching process, in particular the accurate determination of the curve of material strengthening.

The applied Krupkowski fortification rule very well reflects the experimental results. At the beginning of the plastic hardening curve, a small jump is created, which is acceptable, because the pressing processes run in the central part of this curve.

In summary, it can be concluded that the use of a precise measuring system to determine deformations and real stresses is a proper solution in the case of determining the function describing the curve of plastic strengthening.

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